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Macroscopic and mesoscopic matter waves

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Abstract. It has been shown earlier [3,6] that matter waves which are known to lie typically in the range of a few angstrom, can also manifest in the macrodomain with a wave length of a few centimeters, for electrons propagating along a magnetic field. This followed from the predictions of a probability amplitude theory by the author [1,2] in the classical macrodomain of the dynamics of charged particles in a magnetic field. It is shown in this paper that this case constitutes only a special case of a generic situation whereby composite systems such as atoms and molecules in their highly excited internal states, can exhibit matter wave manifestation in macro and mesodomains, in one-dimensional scattering. The wave length of these waves is determined, not by the mass of the particle as in the case of the de Broglie wave, but by the frequency ω, of the classical orbital motion of the internal state in the correspondence limit, and is given by a nonquantal expression, $\lambda = 2\pi v/\omega$, v being the velocity of the particle. For the electrons in a magnetic field the frequency corresponds to the gyrofrequency, Ω and the nonquantal wave length is given by $\lambda = 2\pi v_{\parallel}/\Omega$; $v_{\parallel}$ being the velocity of electrons along the magnetic field.

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1 Introduction

The de Broglie matter waves associated with quantum particles have a wavelength typically a few angstroms ($\lambda = \hbar/mv$) essentially because of the small value of $\hbar$. The question may be asked however, whether matter can exhibit its wave aspect in the macrodimensions as well, not in the sense of macroscopic correlated quantum systems such as superfluids or superconductors, but in the manner and spirit of de Broglie waves associated with single particles. Following the development of the concept of macroscopic matter waves through the theory of reference [1], reinforced by a more recent work by the author [2], we have demonstrated experimentally the existence of such a wave behavior for electrons propagating along a magnetic field having a wavelength independent of $\hbar$ and typically in the range of a few centimeters [3,4,6]. We wish to show here that such a wave manifestation is not entirely peculiar to this system but is a generic property of composite bound systems in their highly excited internal states approaching the classical limit. The wavelength of these new matter waves is related, not to the masses of these particles as in the case of de Broglie waves, but to the frequency associated with their internal state of excitation. This is an entirely new wave manifestation of matter not hitherto pointed out. We also predict the existence of such macroscopic or mesoscopic waves with atoms and molecules.

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There is, however, a qualification: as we shall see, this macro-mesoscopic wave behaviour manifests only in one-dimensional interference. The charged particle dynamics along a magnetic field where such a behaviour has been observed [3,4,6] is inherently one-dimensional (along the magnetic field).

Even though the concept of the macroscopic matter waves in relation to the charged particle dynamics along a magnetic field actually followed from the theories of references [1,2], we present here first a direct quantum mechanical derivation of the macroscopic form of the wave function which is consistent with the form obtained from references [1,2] and which can account for the rather astonishing observations reported [3,4] in this connection, that were predicted by the theory. We later extend these considerations to other systems such as atoms and molecules, and in fact any composite system.

2 Macroscopic wave function and matter waves for charged particles in magnetic field

A charged particle in a magnetic field in the classical mechanical domain corresponds in quantum mechanics to a particle in a Landau level with a very large quantum number. If E_ν be the energy of a Landau level so that

$$E_\nu = \left(\nu + \frac{1}{2}\right)\hbar\Omega, \tag{1}$$

where $\Omega = (eB/mc)$ is the gyrofrequency in the magnetic field B, then $\nu \gg 1$ corresponds to the classical limit and $\nu\hbar = \mu$ defines the gyroaction, which classically has the form $\mu = mv_\perp^2/2\Omega$, ($v_\perp$ is the component of velocity perpendicular to the magnetic field).

Consider now the propagation of an electron beam of a given energy injected into the magnetic field with a small pitch angle δ, so that $v_\perp = v\sin\delta$, and $v_\parallel = v\cos\delta$, $v_\parallel$ being the velocity parallel to the magnetic field. The electrons in this beam are then in a group of Landau levels sharply peaked around the quantum number $\nu = E_\perp/\hbar\Omega = m{v_\perp}^2/2\hbar\Omega$. For a typical laboratory situation, if we choose $E = 1$ keV, and a magnetic field $B = 100$ G, then $\nu \simeq 10^8$, which is clearly $\gg 1$.

A charged particle in a magnetic field can be described by the Hamiltonian

$$H = \frac{{P_\parallel}^2}{2m} + \frac{{P_\perp}^2}{2m} + \frac{1}{2}m\Omega^2\xi^2, \tag{2}$$

where ξ represents the coordinate perpendicular to the magnetic field and the potential energy term $m\Omega^2\xi^2/2$ represents the harmonic oscillator corresponding to the Landau gyro-oscillations, while the first term represents the free motion along the magnetic field. As we shall see later, a similar Hamiltonian also describes the oscillatory motion of a diatomic molecule.

Let χ_ν represent the Landau eigenfunctions which are essentially the harmonic oscillator wave functions [5]. Let there be a scatterer in the path of the electron beam, a small obstacle like the wires in a grid through which the electron beam may be made to pass. The scattering which is assumed to be elastic may kick the electrons from the Landau level ν to $\nu \pm l$, where $\nu \gg l > 1$. If $\tilde{H}$ be the perturbation Hamiltonian which describes the scattering, then the transition amplitude for the process is given by

$$\beta_\nu^{(l)} \equiv \left\langle \nu - l \left| \tilde{H} \right| \nu \right\rangle = \int \mathrm{d}\xi \chi_{\nu-l}(\xi)\tilde{H}\chi_\nu(\xi), \tag{3}$$

where ξ is the coordinate normal to the magnetic field representing the coordinate of the Landau gyro-oscillator.

Let ϕ_ν represent the complete wave function of the particle in a magnetic field including a plane wave corresponding to its free motion along the magnetic field, so that

$$\phi_\nu = \chi_\nu(\xi)\mathrm{e}^{\mathrm{i}\kappa_\nu x}, \tag{4}$$

where

$$\kappa_\nu = \frac{1}{\hbar}\left[2m(E - \nu\hbar\Omega)\right]^{\frac{1}{2}} \tag{5}$$

and x is the coordinate along the magnetic field, while E is the total energy of the particle. The transition amplitude including the eigenfunction along the magnetic field is given by

$$\alpha_\nu^{(l)} = \int \mathrm{d}\xi {\phi^*}_{\nu-l}\tilde{H}\phi_\nu = \beta_\nu^{(l)} \exp\left[\mathrm{i}\left(\kappa_\nu - \kappa_{\nu-l}\right)x\right]. \tag{6}$$

Now making use of the assumption $l \ll \nu$, we expand $\kappa_{\nu-l}$ around κ_ν using the expression (5) which gives

$$\kappa_\nu - \kappa_{\nu-l} \simeq l\frac{\partial\kappa_\nu}{\partial\nu} = \frac{l\Omega}{v}, \tag{7}$$

where v is the velocity of the particle along the magnetic field

$$v = \left[\frac{2}{m}\left(E - \nu\hbar\Omega\right)\right]^{1/2}. \tag{8}$$

The difference $(\kappa_\nu - \kappa_{\nu-l}) = l\Omega/v$ represents the change in the wave number κ_ν of the plane wave, in consequence of the change in the Landau level quantum number from ν to $\nu - l$ due to the elastic scattering off the obstacle. The transition amplitude is thus given by

$$\alpha_\nu^{(l)} = \beta_\nu^{(l)} \exp\left[\mathrm{i}(l\Omega/v)x\right]. \tag{9}$$

This transition amplitude is again a wave function, representing a plane wave by virtue of the exponential factor. But as we notice, it is clearly independent of $\hbar$. Since it is derived directly from the quantum mechanical wave function (4), it represents a matter wave with a wave length $\lambda_l = 2\pi v/l\Omega$. For an electron energy parallel to the magnetic field $E_\parallel = 500$ eV and a magnetic field $B = 100$ G, we find $\lambda_1 \simeq 5$ cm, for $l = 1$. Thus this matter wave length falls in the macrodomain in contrast to the usual de Broglie wave length which is generally in the Å range.

It may be mentioned that these ideas have been more formally expressed in a recent paper by the author [2] where he has derived a set of one-dimensional Schrödinger-like equations starting from the quantum mechanic Schrödinger equation (in its path integral representation) for the charged particle dynamics in a magnetic field. These are

$$\frac{\mathrm{i}\mu}{l}\frac{\partial\Psi(l)}{\partial t} = -\left(\frac{\mu}{l}\right)^2\frac{\partial^2\Psi(l)}{\partial x^2} + \big(\mu\Omega\big)\Psi(l), \quad l = 1, 2, \ldots \tag{10}$$

where x is the coordinate along the magnetic field, and where μ, which is the gyroaction and has been shown in typical laboratory conditions to be $\sim 10^8\hbar$, is a classical object and appears in the role of $\hbar$ in these equations. In terms of the notation of the foregoing treatment $\mu = \nu\hbar$ with $\nu \gg 1$. Furthermore, the wave functions $\Psi(l)$ of these equations are actually the transition amplitudes as defined above, from the quantum mechanical state with a large Landau quantum number ν to the one with quantum number $\nu - l$, induced by a perturbation. The number l labels this wave function as $\Psi(l)$.

It is worth remarking that by virtue of the large quantum number ν ($\sim 10^8$) of the basic state involved and the small quantum number interval l across which transitions are envisaged, these equations for the amplitude functions $\Psi(l)$ describe the dynamics of the system in the correspondence limit. This is the classical limit in accordance with correspondence principle of Bohr. Thus by virtue of the large (classical) value of μ which appears in the place of $\hbar$, these equations for the amplitude functions $\Psi(l)$ describe one-dimensional (along the magnetic field) matter wave phenomena in the macrodomain of classical mechanics. This is essentially equivalent to what has been demonstrated above in equation (9), in more direct manner.

The wave function of equation (9), as also the Schrödinger-like equations (10) predict the matter wave

phenomena with the wave length of a few centimeters for the charged particles moving along a magnetic field. The one-dimensional matter wave interference phenomena which correspond to these macroscopic wave functions of the form (9) have indeed been observed by the author and his coworkers [3,4,6].

The experimental results reported in references [3,4] exhibit the existence of discrete energy bands (the maxima and minima) in the transmission of electrons along a magnetic field, when the electron energy from an electron source is swept as they transit from the latter along a magnetic field to a detector plate a distance L_p away. These bands which are rather unexpected in the parameter domain of the experiments where classical mechanical equations of motion are supposed to operate, have been identified as the interference maxima and minima in the energy domain, with a (nonquantal) macroscopic wave length $\lambda = 2\pi v_{\|}/\Omega$, in accordance with the form (9) of the macroscopic wave function. The interpeak separation of the transmission bands (in energy) are found to be inversely proportional to the distance L_p, so that the latter corresponds to a frequency as the energy is swept. The experiments of references [3,4] thus confirm the predictions of the theory on the existence of the macroscopic form of the matter waves (see however, comments appended to Ref. [3] of the list of references).

We have also found the existence of beat phenomena [6] in these experiments – a modulating beat structure of the already reported discrete energy band structure when the two "frequencies" in the system are close together. In the presence of a grounded grid at a distance L_g from the electron gun, one has two frequencies in the system corresponding to the L_p and L_g and the frequency of the observed beats in the transmitted signal is found to correspond to the difference $(L_p - L_g)$, where $(L_p - L_g) \ll L_p$. This is just what occurs in other wave phenomena as well. These observed beats thus constitute a further, even tighter evidence for the wave behaviour of particles moving along a magnetic field.

3 Macroscopic matter waves for composite systems in their high internal state of excitation

Having discussed the concept of macroscopic matter waves for charged particles in a magnetic field in the last section, whose wave manifestations in the macrodomain have also been observed, we now extend these considerations to other composite systems such as atoms and molecules in their internal state of excitation. We first discuss some gedanken experiments to point out the possible macroscopic wave manifestations of these systems and then discuss the possibility of carrying out real experiments to observe these manifestations associated with such composite systems.

3.1 Diatomic molecule in a highly excited vibrational state

First we consider a diatomic molecule in a highly excited vibrational state ignoring for the moment its rotational and electronic degrees of freedom. Such a system is described by a Hamiltonian similar to the one given by (2) where the one-dimensional "parallel" momentum $p_{\|}$ is replaced by the three-dimensional momentum $\mathbf{P}$ of the centre of mass M of the diatomic molecule and the "perpendicular" momentum $p_{\perp}$ is replaced by the momentum $\mathbf{p}$ of the reduces mass m, identifying ξ as the reduced mass coordinate. We then have the Hamiltonian as

$$H^{v}_{\mathrm{DA}} = \frac{P^2}{2M} + \frac{p^2}{2m} + \frac{1}{2}m\omega^2\xi^2 \tag{11}$$

where we have now the vibrational frequency ω of the diatomic molecule, and we have of course ignored the anharmonic terms for simplicity. We shall comment later on the effects of the anharmonic terms.

If we employ a similar notation as before then the eigenfunction for the system with the Hamiltonian H^{v}_{DA} corresponding to the free motion of the centre of mass with momentum $\mathbf{P}$, and vibrational state ν is given by

$$\psi(\mathbf{P},\nu) = A_1 \mathrm{e}^{\mathrm{i}\mathbf{P}\cdot\mathbf{X}/\hbar}\chi_\nu(\xi), \tag{12}$$

with the total energy E

$$E = \frac{P^2}{2M} + \hbar\omega\left(\nu + \frac{1}{2}\right), \tag{13}$$

where $\chi_\nu(\xi)$ are the normalized harmonic oscillator wave functions.

Consider now a beam of such particles with a given momentum $\mathbf{P}$ and in a highly excited vibrational state $\nu \gg 1$, which can be prepared using appropriate laser techniques. Let the beam be scattered by a grid of scatterers G_1 at the point $\mathbf{X}_1$ in its path, with small transverse dimensions. Assume that the scattering is elastic with respect to the total energy E of the particle, and the scattering changes only its internal vibrational state to ν', so that we have from the energy conservation

$$E = \frac{P^2}{2M} + \nu\hbar\omega = \frac{P'^2}{2M} + \nu'\hbar\omega \tag{14}$$

where $\mathbf{P}'$ is the centre of mass momentum after the scattering. Thus the final state after the scattering is

$$\Psi' = A_1' \mathrm{e}^{\mathrm{i}\mathbf{P}'\cdot(\mathbf{X}-\mathbf{X_1})/\hbar}\chi_{\nu'}(\xi) \tag{15}$$

where $|\nu' - \nu| \ll \nu$. If then $\tilde{H}(\xi)$ is the perturbing Hamiltonian which causes the scattering, then the transition amplitude at the point $\mathbf{X}$ is given by

$$\alpha_{\nu'\nu} = \langle\nu'|\tilde{H}|\nu\rangle = A_1 A_1' \exp\left[-\mathrm{i}\left(\mathbf{P}' - \mathbf{P}\right)\cdot(\mathbf{X} - \mathbf{X_1})/\hbar\right] \times \int \mathrm{d}\xi \chi_{\nu'}(\xi)\tilde{H}(\xi)\chi_\nu(\xi) \tag{16}$$

where A_1 and A_1' are appropriate normalization constants.

We consider a one-dimensional situation, whereby we choose the scattered momentum $\mathbf{P}'$ to be in the same direction as the initial momentum $\mathbf{P}$ and to take the X-axis to be in the same direction as well. Then

$$(\mathbf{P}' - \mathbf{P}) \cdot (\mathbf{X} - \mathbf{X}_1) = (P' - P)(X - X_1). \tag{17}$$

Using the energy conservation (14), we have $(P' - P) = 2M\hbar\omega(\nu - \nu')/(P' + P)$. If we write $\nu - \nu' = l$, where $l \ll \nu$, then the change in the internal energy $\Delta E_i = l\hbar\omega \ll P^2/2M$. One may then approximate $P + P' \simeq 2P$, so that $(P' - P) \simeq l\hbar\omega/v$, where $v = P/M$ is the velocity of the centre of mass which is identified as the beam velocity assumed to be sharply peaked at v. Using this in (16) yields the transition amplitude as

$$\alpha^{(1)}_{\nu'\nu}(X) = A_1 A_1' \beta_{\nu'\nu} \exp\left[\mathrm{i}\frac{l\omega}{v}(X - X_1)\right], \tag{18}$$

where $\beta_{\nu'\nu}$ is the matrix element of the perturbation $\tilde{H}(\xi)$ between the oscillator states ν' and ν. The form of the perturbation $\tilde{H}$ is left general enough, and may be given any specific form as required. The important thing to note is that the transition amplitude for the translational centre of mass degree of freedom has the form $\exp[\mathrm{i}(l\omega/v)X]$ and is independent of $\hbar$. Note that in general the scattering at the grid G_1 could lead to different values of $l = 1, 2, 3...$

Consider next another grid G_2 of scatterers located at the point X_2 in the path of the beam. The scattering (transition) amplitude from this grid is given by

$$\alpha^{(2)}_{\nu'\nu}(X) = A_2 A_2' \beta_{\nu'\nu} \exp\left[\frac{\mathrm{i}l\omega}{v}(X - X_2)\right], \tag{19}$$

where A_2 and A_2' are again appropriate normalization constants.

Note that the expressions (18, 19) represent wave functions corresponding to a wave number $k_l = (l\omega/v) = lk$ which is the lth harmonic of the basic wave number $k = \omega/v$, the corresponding wave length being $\lambda = 2\pi v/\omega$. This is clearly independent of $\hbar$, and could lie in the macro or meso-domain. For a typical diatomic molecule, the vibrational wave number is $(\omega/2\pi c) \approx 2 \times 10^3\ \mathrm{cm}^{-1}$. Taking a modest value of beam velocity $v \approx 10^8\ \mathrm{cm\,s}^{-1}$, this gives $\lambda \approx 0.1\mu$. This is about three orders of magnitude larger than the typical de Broglie wave length of a few Å.

One can now look for interference between the waves given by (18, 19) originating at the scatterer grids at X_1 and X_2. At a point X downstream of the grids at X_1 and X_2 the total amplitude is given by

$$\begin{aligned}\alpha &= \alpha^{(1)}_{\nu'\nu} + \alpha^{(2)}_{\nu'\nu} \\ &= \mathrm{e}^{\mathrm{i}lkX} \beta_{\nu'\nu} \left[A_1 A_1' \mathrm{e}^{-\mathrm{i}lkX_1} + A_2 A_2' \mathrm{e}^{-\mathrm{i}lkX_2}\right]\end{aligned} \tag{20}$$

whence the intensity of the scattered particles is given by

$$\begin{aligned}|\alpha|^2 = |\beta_{\nu'\nu}|^2 \Big\{&(A_1 A_1')^2 + (A_2 A_2')^2 \\ &+ 2\,(A_1 A_1' A_2 A_2') \cos[lk\,(X_1 - X_2)]\Big\} \cdot\end{aligned} \tag{21}$$

This therefore describes interference maxima and minima through the $\cos[kl\,(X_1 - X_2)]$ term which is independent of $\hbar$ and hence belongs to a nonquantal domain. Such interference effects should be present in the experimental arrangement described above. This is analogous to the double slit interference, the grids at X_1 and X_2 corresponding to the two slits, but now in one dimension. One can check the validity of the expression (21), by working with different diatomic molecules to vary ω, and different beam velocities to check the dependence on v, as well as different values of $(X_1 - X_2)$.

Note that there would, in general, exist many values of $l = 1, 2, 3...$ in the excitation spectrum of the vibrational states of the diatomic molecule as a result of scattering off the grids G_1 and G_2. However, $l = 1$ would be the most dominant one, being closest in energy to the central quantum number ν. There would thus exist interference maxima, for the fundamental $l = 1$. When the higher harmonic terms corresponding to $l = 2, 3, ...$ are included, they would lead to a change of shape of the peaks corresponding to the fundamental as the expression (21) is summed over l with appropriate weights. Higher harmonics will lead to appropriate change of shape of the peaks corresponding to the fundamental. Before we proceed to discuss other cases, some comments on this case are in order: first, we recall that we had idealized the treatment of this case by ignoring the anharmonicity in the vibrational motion of the molecule. This would be justified if the initial prepared state peaked around a vibrational quantum number ν_o lies well within the harmonic regime. The latter regime has the well-known feature that the energy difference between the neighbouring levels is independent of the level quantum number. This circumstance enables one to choose the initial state to be a low lying state, which has the advantage of being far away from the region of anharmonicity. One may also add parenthetically that the same property gives one the flexibility in this case of not preparing a highly peaked state.

A comparison may be made, as was mentioned earlier, between the diatomic molecule in its vibrational state and the charged particle in a magnetic field which was discussed in Section 2. Both are bound systems. The latter is bound harmonically normal to the magnetic field, while it has a free motion along the field direction. There is no anharmonicity in this case, and the free motion pertains to just one of its degrees of freedom along the magnetic field. In the case of the diatomic molecule, it is the reduced mass of the system which is bound, while the free motion pertains to its centre of mass. These differences notwithstanding, the nature of the one-dimensional macro-mesoscopic interference phenomena is the same, with the matter wave length given by similar expressions: $\lambda_\mathrm{ch} = 2\pi v_\parallel/\Omega$, ($\Omega = eB/mc$, the gyrofrequency, and $v_\parallel$, the velocity parallel to the magnetic field), for the particle in a magnetic field and $\lambda_\mathrm{DV} = 2\pi v/\omega$, ($\omega$, the frequency of oscillation of the molecule, and v the velocity of its centre of mass), for the diatomic molecule with respect to its vibrational state.

3.2 Rotational and vibrational states of a diatomic molecule

We next consider the case of the rotational states of the diatomic molecule which belong to a Hamiltonian different from (11). The energy of its rotational state with the angular momentum quantum number j is $E_j = K\hbar^2 j(j+1) \simeq K\hbar^2 j^2$ for highly excited states. If we again assume the scattering off the grids to be elastic with respect to the total energy of the molecule, we have

$$\frac{P^2}{2M} + K\hbar^2 j^2 = \frac{P'^2}{2M} + K\hbar^2 j'^2, \tag{22}$$

whence

$$(P - P') = \frac{2MK\hbar^2(j + j')(j' - j)}{(P + P')} \simeq \frac{2K\hbar j l_j}{v}\ , \tag{23}$$

where we have assumed $l_j = (j' - j) \ll j$, whence $(P + P') \simeq 2P$ follows. We now use an expression similar to (16) for the transition amplitude $\alpha_{j'j}$ involving the transition from the rotational state j to j', and taking the scattering to be one-dimensional as before,

$$\alpha_{j'j} = A_1 A_1' \gamma_{j'j} \exp\left[-\mathrm{i}(P' - P)(X - X_1)/\hbar\right], \tag{24}$$

where $\gamma_{j'j}$ is the matrix element of the perturbation Hamiltonian $\tilde{H}(\vartheta)$ between the rotational states, $\Theta_{j'm}$ and Θ_{jm}

$$\gamma_{j'j} = \int \mathrm{d}\vartheta \mathrm{d}\varphi \Theta_{j'm} \tilde{H}(\vartheta) \Theta_{jm}. \tag{25}$$

We note that the rotational energy of the diatomic molecule is degenerate with respect to the quantum number m. If we now make use of (23) to substitute for $(P - P')$ in (24), we obtain for the transition amplitude $\alpha_{j'j}$

$$\begin{aligned}\alpha_{j'j} &= A_1 A_1' \gamma_{j'j} \exp\left[\mathrm{i} l_j \frac{2KJ}{v}(X - X_1)\right] \\ &= A_1 A_1' \gamma_{j'j} \exp\left[\mathrm{i}\frac{l_j \omega_j}{v}(X - X_1)\right],\end{aligned} \tag{26}$$

where we have introduced $J = j\hbar$ which in the limit of large j is a classical object, the angular momentum of the molecule. Also $KJ = J(2mR^2)^{-1}$, m being the reduced mass, and R the internuclear distance. Then $2KJ = J/mR^2 = \omega_j$, where ω_j is the angular velocity corresponding to the quantum number j and the angular momentum $J = \hbar j$. The amplitude is again of the form (18) and involves ω_j and v which may be regarded as classical objects, and is thus independent of $\hbar$ and corresponds to a wave length of meso or macrodimension. However, the important difference between this case and the vibrational case is that while ω in the latter case is independent of the quantum state, ω_j increases linearly with the quantum number j. However, for large j it can be regarded as almost a constant over a small range Δj of j.

In general, however, the scattering by the grid of scatterers would induce transitions in vibrational as well as in rotational states from the initially prepared state. By following the arguments used earlier it can be shown that one would then have a general wave amplitude of the form:

$$\begin{aligned}\alpha_{j'j\,;\,\nu'\nu} &= A_1 A_1' \Gamma_{j'j\ \nu'\nu} \\ &\quad \times \exp\left[\frac{\mathrm{i}}{v}\left(l_\nu \omega_\nu + l_j' \omega_j\right)(X - X_1)\right],\end{aligned} \tag{27}$$

where now we mean by ω_ν, the vibrational frequency and by ω_j, the rotational frequency, and v is again the velocity of the centre of mass. We define the corresponding wave numbers as $k_\nu = \omega_\nu/v$ and $k_j = \omega_j/v$. We thus have a more general expression for the intensity of the superposed waves scattered off the grids at X_1 and X_2, involving both the vibrational and rotational transitions

$$\begin{aligned}|\alpha|^2 = |\Gamma_{j'j\nu'\nu}|^2 \Big\{&(A_1 A_2')^2 + A_2 A_2')^2 \\ &+ 2(A_1 A_1' A_2 A_2') \cos\left[(l_\nu k_\nu + l_j k_j)(X_1 - X_2)\right]\Big\},\end{aligned} \tag{28}$$

where it may be remembered that the vibrational frequency of a diatomic molecule $\omega_\nu \gg \omega_j$, the rotational frequency. Recall also that l_ν and l_j represent level differences respectively of the vibrational and rotational states across which transitions occur. Both l_ν and l_j would have a range of values, both positive and negative, which the expression (28) will be summed over with different weight factors corresponding to l_ν and l_j. Since the latter represent harmonic numbers of the frequencies ω_ν and ω_j, the two exponential factors with these frequencies (when the cosine term is expressed in terms of the exponential factors) would yield a periodic function F_ν of $k_\nu(X_1 - X_2)$ multiplied by a periodic function F_j of $k_j(X_1 - X_2)$, with their shapes determined by the weights of l_ν and l_j respectively

$$\sum_{l_\nu, l_j} |\alpha_{\nu'\nu, j'j}|^2 \sim F_\nu\left[k_\nu(X_1 - X_2)\right] F_j\left[k_j(X_1 - X_2)\right]. \tag{29}$$

Since $k_j \ll k_\nu$ (as $\omega_\nu \gg \omega_j$), the slower variation of the function F_j with $(X_1 - X_2)$ or with v for a fixed $(X_1 - X_2)$ will amplitude modulate the rapid variation of F_ν with $(X_1 - X_2)$, with the maxima of the modulation separated on the X-scale by a distance $L_j = 2\pi/k_j$, while the maxima of the rapid variation will be separated by the distance $L_\nu = 2\pi/k_\nu$. This is what one would expect to observe experimentally.

3.3 Rydberg states of an atom

One may also consider the Rydberg states of an atom for a similar discussion, which presents a rather interesting case. The energy levels for this case are given by

$$E_n = -\frac{me^4}{2\hbar^2 n^2}.$$

Using the expressions $P = \left[2M\left(E - E_n\right)\right]^{1/2}$ and $P' = \left[2M\left(E - E_{n'}\right)\right]^{1/2}$, we find $P - P' \approx (2lE_n/nv)$ where

$v = [2(E - E_n)/M]^{1/2}$ is again the magnitude of the centre of mass velocity with the total energy being E, and where $l = |n' - n| \ll n$ is assumed. Using the foregoing expression for $P' - P$ in the expression corresponding to (18) gives

$$\alpha_{n'n} = A_1 A_1' \, \Delta_{n'n} \exp\left[\mathrm{i}\frac{2lE_n}{nv\hbar}(X - X_1)\right], \qquad (30)$$

where $\Delta_{n'n}$ is the scattering (transition) amplitude for a perturbation $\tilde{H}$ between the Rydberg states n' and n. Expression (30), of course, involves $\hbar$ unlike the expressions (18, 19) for the vibrational degree of freedom, which are independent of $\hbar$. However, $\hbar$ appears in the denominator of the exponent in equation (30) in the combination $n\hbar$. Thus when n is large enough, $J_n = n\hbar$ may be regarded as a classical object, namely the angular momentum of the nth Rydberg state in the correspondence limit. We therefore write

$$\frac{2E_n}{v(n\hbar)} = \frac{2E_n}{vJ_n} = \frac{\omega_n}{v}, \qquad (31)$$

where we introduce $\omega_n = 2E_n/J_n$, which can be shown to be the frequency of the classical orbit (in the correspondence limit). For large n this frequency may be considered as almost a constant for small variations of n. Equation (30) is then

$$\alpha_{n',n} = A_1 A_1' \exp[\mathrm{i}k_n(X - X_1)], \qquad (32)$$

where $k_n = \omega_n/v$. We notice that $k_n \sim n^{-3}$, and thus decreases rapidly with n, and would correspond to mesoscopic matter waves if n is sufficiently large. For $n = 100$, $\omega_n \approx 6.6 \times 10^{10}$ rad s^{-1}, and $k_\parallel \approx 660$ cm^{-1} for $v = 10^8$ cm s^{-1}. This gives $\lambda_n = 2\pi/k_n \approx 10^{-2}$ cm, which is of macroscopic, or if one prefers, of mesoscopic dimensions.

4 Suggested experimental arrangement

There are the following four main elements which constitute the basic requirements of the experiment to check the existence of mesoscopic matter waves proposed here. One could choose either a diatomic molecule or a Rydberg atom depending on the overall convenience. There would of course arise a number of technical problems during the course of carrying out the experiment which will need to be addressed. We shall discuss here only broadly the various essential requirements and a possible experimental arrangement.

1. First, it will be necessary to prepare the molecules/atoms in an appropriate highly excited state. As has been already discussed, because of the special features of the harmonic potential, even low lying vibrational states can be selected for state preparation for the diatomic molecule. For the rotational states, on the other hand it would be desirable to have a highly excited state, for only in that case ω_j which equals $2Kj\hbar$ would be nearly constant for small variations of j when j is large. Such states can be prepared with appropriate laser techniques and some ingenuity. A Rydberg atom can also be similarly prepared in a highly excited state so that its frequency $\omega_n = 2E_n/n\hbar$ is nearly constant for small variations of n when n is large.
2. It will be necessary next to prepare a beam of such molecules/atoms with a well defined velocity. A velocity of 10^8 cm s^{-1} corresponds roughly to an energy, $\mathcal{E} \sim 10^4$ eV which should be possible without much difficulty using an ion source.
3. Another requirement is the fabrication of grids for the scattering of molecules/atoms as envisaged above. The distance $(X_1 - X_2) = D$ between the grids would be required to be of the order of a few wave lengths ≤ 10 (of the particular matter wave). However, to obtain a higher resolution it would be desirable to have an array of 5–10 grids. While a larger number of grids would yield higher resolution, it will also lead to a decreasing transparency for the transmitted signal. So, the number of grids in the array will have to be optimized. This will be similar to crystal lattice planes in electron diffraction. Here the grid array will have to be properly fabricated. These will be required to be rigid and rigidly fixed to the apparatus so as to ensure the elastic nature of the scattering with respect to the total energy of the molecules/atoms.
4. Finally one has to ensure the quasi-one-dimensionality of the scattering process, which is the most crucial requirement for the manifestation of the mesoscopic matter waves. One way to ensure is to use a beam of particles with a broad wave front parallel to the plane of the grids. This is similar to the Bragg diffraction with the electron beam normal to the plane of the crystal. However, a more definitive way to ensure it in the present case is to use charged molecules/atoms in a given charge state injected along an ambient magnetic field. This will constrain the molecules to move along the one-dimension of its direction. The field may be taken to be at any angle θ to the plane of the grid array, and defines the direction of the one-dimensional scattering. This arrangement has the added advantage that the intergrid spacing along the direction of the magnetic field, which is the direction of the one-dimensional scattering, can be varied by varying the angle θ. Thus the effective intergrid distance $D_{\mathrm{eff}} = D/\sin\theta$.

It ought to be mentioned, however, that since the thickness of the grid wires would be roughly of micron size or less (a few hundred nanometer diameter wires are supposed to be available now) much larger than the dimension of the molecules, the forward scattering off the grid wires envisaged above would have to be tangential. An efficient scattering would then require that there be larger number of wires of as small a thickness as possible. If χ be the grazing angle which the incident molecule makes with the tangent plane to the grid wire, then it can be easily shown that even if the grid wire of mass M_G were free, then the energy of recoil acquired by it in such

a collision with the molecule of mass M, moving with the velocity v would be $\sim (M^2/M_G)v^2 \sin^2\chi$, which is negligibly small in the ratio (M/M_G). Thus the elasticity of collisions with the the grid wires with respect to the total energy of the molecule is well satisfied. However, if the grid wires are rigid and rigidly fixed to the apparatus then elasticity is satisfied *a fortiori*.

5 Summary and discussion

The concept of macroscopic and mesoscopic matter waves as distinct from the de Broglie matter waves that has been presented above, first arose in relation to the dynamics of charged particles in a magnetic field as described by a probability amplitude theory, applicable in the macrodomain, and represented by a set of one-dimensional Schrödinger-like equations obtained by the author [1,2]. The role of $\hbar$ in these equations is enacted by a macroscopic action $\mu = \nu\hbar, (\nu \gg 1)$ as a result of which they predicted the existence of macroscopic matter wave interference phenomena. The wave length of these waves is found to be given by $\lambda = 2\pi v_\parallel/\Omega$, where $v_\parallel$ is the velocity parallel to the magnetic field, and $\Omega = eB/mc$ is the gyrofrequency. This expression is, however, obtained in Section 2 by a direct quantum mechanical derivation. For typical laboratory conditions: electron energy ~ 1 keV and magnetic field, $B = 100$ G, $\lambda \approx 5$ cm. Such macroscopic interference phenomena have indeed been observed [3,4,6].

An examination of the derivation of Section 2 showed that similar arguments could be applied to the highly excited vibrational-rotational state of a diatomic molecule pertaining to the correspondence limit. The latter limit ensures that in any small range Δn of quantum numbers in this limit ($\Delta n \ll n$), the levels are equidistant with the interval being $\hbar\omega$, where ω is the classical orbital frequency $\omega = 2\pi(\oint \mathrm{d}q/v_q)^{-1}$, in the region of large quantum number n, where v_q is the classical orbital velocity (see, for example Ref. [5], p. 165). This is as if a system (any system) behaves like a harmonic oscillator over a small range $\Delta n \ll n$ in the correspondence limit. This result is true for any system in the correspondence limit, including the Rydberg atom, besides the diatomic molecule in its vibrational-rotational state.

It has been demonstrated here that such composite systems should exhibit mesoscopic matter wave interference, in a one-dimensional scattering off an array of grids, with a wave length given by $\lambda = 2\pi v/\omega$, where ω is the classical orbital frequency of the internal motion, and v, the velocity of the centre of mass. In this form it is quite a generic expression for the macroscopic matter wave length, which is independent of the Planck quantum and involves only the classical quantities ω and v, but not the mass of the particle, and yet represents a matter wave.

Clearly this generic expression arises through the characteristic property of the correspondence limit as given above. The mesoscopic wave manifestation may thus be regarded as a persistence of the matter wave behaviour into the classical domain, though admittedly it is restricted to one-dimensional case only. This can be regarded as a classical counterpart of the quantum de Broglie wave. This however is no violation of the correspondence principle as we understand it. In fact the above discussion makes use of it. On the other hand, if by correspondence principle one were to mean that quantum mechanics should go over into classical mechanics without exception in the limit of large quantum numbers, then surely the results reported here show that it is not entirely true.

It may, however, be mentioned that historically the correspondence principle concerned itself with the problem relating to the frequency of radiation from an atom; stating that in the region of large quantum numbers when the levels become almost equally spaced, the quantum frequencies approach the classical orbital frequency which is identified as the frequency of radiation classically. Questions relating to the transition amplitudes between different levels in the correspondence limit were perhaps not posed (to the best of author's knowledge). It is the properties of these transition amplitudes which have revealed the macro-mesoscopic wave phenomena when energy exchange is taken into account between the internal states and the centre of mass motion in one-dimensional scattering. Equations (10) are, in fact (as already mentioned) precisely the equations governing these transition amplitudes for charged particles in a magnetic field. Such situations had not been considered before.

Finally, we suggest experiments that can be carried out to check these predictions, and discuss in Section 4 some aspects relating to their basic requirements. This discussion relating to the experiments is meant to be only indicative and not at all exhaustive. Clearly there would arise a number of technical problems during the course of the experiment which will have to be suitably addressed.

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which is such that the electron source whose energy was swept as a part of the experimental procedure, was here a secondary electron source. It has been discussed in detail in reference [6] that their mechanism would not work for the results on the discrete energy bands and beats reported therein (Ref. [6] because of a different and simpler methodology used). In fact the beat frequency observed is entirely in accordance with the wave picture as already mentioned in the text

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